Embedding Intensional Semantics into Inquisitive Semantics

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Semantics of interrogative clauses, e.g.

- *Does Mary sleep?*
- *Who sleep?*
- *John knows whether Mary sleeps.*

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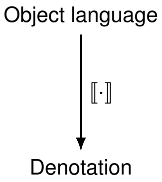
- Object language: simply typed λ-calculus
- Denotation: sets and functions
- Map: **compositional interpretation**



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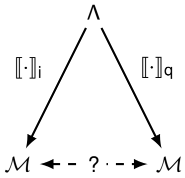

We investigate the relation between:

Intensional interpretation $[\![\cdot]\!]_i$ (declarative)

vs. **Inquisitive interpretation** $[\![\cdot]\!]_q$ (declarative + interrogative)

Object language

Simply typed λ-calculus

- atomic types:
  - PROP : proposition
  - IND : individual

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  - $\textsc{prop}$: proposition
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- linguistic constants
  - Mary: $\mathbf{m} : \textsc{ind}$, John: $\mathbf{j} : \textsc{ind}$, Ash: $\mathbf{a} : \textsc{ind}$
  - $\mathbf{sleep} : \textsc{ind} \to \textsc{prop}$
  - $\mathbf{try} : \textsc{ind} \to (\textsc{ind} \to \textsc{prop}) \to \textsc{prop}$
- logical connectives
  - $\neg : \textsc{prop} \to \textsc{prop}$
  - $\wedge : \textsc{prop} \to \textsc{prop} \to \textsc{prop}$
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- Example:

(1) Mary sleeps.

$$\mathbf{sleep}\ \mathbf{m} : \textsc{prop}$$

(2) Mary does not sleep.

$$\neg(\mathbf{sleep}\ \mathbf{m}) : \textsc{prop}$$

(3) Mary tries to sleep.

$$\mathbf{try}\ \mathbf{m}\ (\lambda x.\ \mathbf{sleep}\ x) : \textsc{prop}$$

Intensional interpretation

Intensional model:

- truth values $\{0, 1\}$
- individuals $D = \{m, j, a\}$
- possible world $W = \{\mathrm{M}, \mathrm{J}, \mathrm{A}\}$

World	Mary sleeps	John sleeps	Ash sleeps
M	yes	no	no
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$$[\![\mathbf{sleep\ m}]\!]_{\mathrm{i}} = [\![\mathbf{sleep}]\!]_{\mathrm{i}}\,([\![\mathbf{m}]\!]_{\mathrm{i}}) = \{\mathrm{M}\}$$
$$[\![\neg\,(\mathbf{sleep\ m})]\!]_{\mathrm{i}} = W \setminus [\![\mathbf{sleep\ m}]\!]_{\mathrm{i}} = \{\mathrm{J}, \mathrm{A}\}$$

Semantics of questions

(4) Does Mary sleep?

Request of information, 2 (maximal) resolutions:

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Hamblin's **alternative semantics** [4]:

- interrogative proposition = **set of answers** (i.e. intentional proposition), e.g.

$$\begin{aligned}[\![\text{Does Mary sleeps?}]\!]_q &= \{[\![\text{Mary sleeps.}]\!]_i\ ,\ [\![\text{Mary does not sleep.}]\!]_i\ \} \\ &= \{\{\mathsf{M}\}, \{\mathsf{J}, \mathsf{A}\}\}\end{aligned}$$

- several drawbacks [2]

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Representing a question by the **downward-closed** set of its answers:

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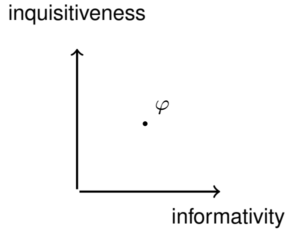
- general notion: **issue**
- 2 dimensions:



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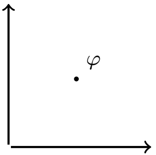
inquisitiveness



informativity

- purely informative issue : only 1 alternative, e.g.

$$[\![\text{Mary sleeps.}]\!]_q = \{\{\mathsf{M}\}\}$$

→ uniform account of declaratives and interrogatives

Question

Suppose we have a complete intensional lexicon

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Practical need

Can we design an inquisitive interpretation **out of** the intensional one?

Inquisitive interpretation

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$$\begin{aligned} [\![\mathbf{m}]\!]_{\mathsf{q}} &= m \in D \text{ same for } \mathbf{j}, \mathbf{a} \\ [\![\mathbf{sleep}]\!]_{\mathsf{q}} &= d \mapsto \mathscr{P}([\![\mathbf{sleep}]\!]_{\mathsf{i}}(d)) \\ [\![\mathbf{try}]\!]_{\mathsf{q}} &= d, P \mapsto ??? \end{aligned}$$

$$\text{with } P : D \to \mathscr{P}(\mathscr{P}(W))$$
$$\text{but } [\![\mathbf{try}]\!]_{\mathsf{i}} : D \to (D \to \mathscr{P}(W)) \to \mathscr{P}(W)$$

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What we want:

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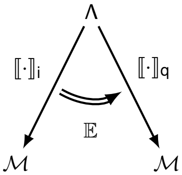
$\rightarrow$ Conservative extension : preserves the logic

Inquisitivation:

- Embedding intentional semantics into inquisitive semantics



First ideas

$\mathbb{E}$ indexed by types of Λ, following [3]

$$\mathbb{E}_{\mathsf{IND}} : D \to D$$
$$\mathbb{E}_{\mathsf{PROP}} : \mathscr{P}(W) \to \mathscr{P}(\mathscr{P}(W))$$
$$\mathbb{E}_{\alpha} : [\alpha]_{\mathsf{i}} \to [\alpha]_{\mathsf{q}}$$

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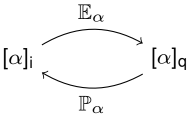
On simple examples:

$$\mathbb{E}_{\mathrm{IND}}(d) = d$$
$$\mathbb{E}_{\mathrm{PROP}}(p) = \mathscr{P}(p)$$
$$\mathbb{E}_{\mathrm{IND}\to\mathrm{PROP}}(P) = d \mapsto \mathscr{P}(P(d))$$

Embedding and projection

- **Projection** $\mathbb{P}$ and **embedding** $\mathbb{E}$ defined by mutual induction



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$$[\alpha]_\mathsf{i} \xrightarrow{\ \mathbb{E}_\alpha\ } [\alpha]_\mathsf{q} \qquad [\alpha]_\mathsf{q} \xrightarrow{\ \mathbb{P}_\alpha\ } [\alpha]_\mathsf{i}$$

Using

$$\bigcup : \quad \begin{array}{rcl} \mathscr{P}(\mathscr{P}(W)) & \rightarrow & \mathscr{P}(W) \\ \mathcal{I} & \mapsto & \bigcup_{p \in \mathcal{I}} p \end{array}$$

Property:

$$\bigcup(\mathscr{P}(p)) = p$$

Definition of inquisitivation

Complete definition:

$$\begin{aligned}
\mathbb{E}_{\mathsf{IND}}(d) &= d \\
\mathbb{E}_{\mathsf{PROP}}(p) &= \mathscr{P}(p) \\
\mathbb{E}_{\alpha\to\beta}(f)(a) &= \mathbb{E}_{\beta}(f(\mathbb{P}_{\alpha}(a)))
\end{aligned}$$

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Example:

$$[\![\mathbf{try}]\!]_{\mathsf{q}}\,(d)(P) = \mathbb{E}_{\mathsf{IND}\to(\mathsf{IND}\to\mathsf{PROP})\to\mathsf{PROP}}([\![\mathbf{try}]\!]_{\mathsf{i}}\,)(d)(P)$$

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Inquisitive interpretation of a constant $c : \alpha$

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Formulas:

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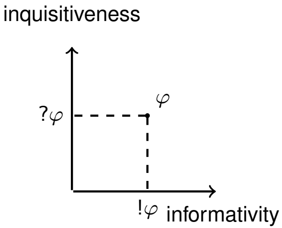

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Same intensional model $\langle D, W, \mathcal{I}\rangle$
But operations on sets of sets:

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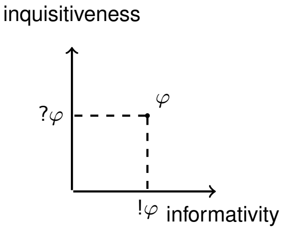

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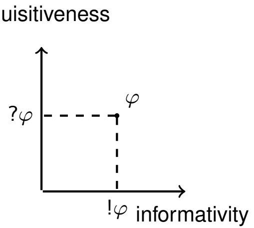

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[\![\neg\varphi]\!] &= \{s \mid \forall t \in [\![\varphi]\!] \,.\, s \cap t = \varnothing\} \\
[\![\varphi \wedge \psi]\!] &= [\![\varphi]\!] \cap [\![\psi]\!] \\
[\![\varphi \vee \psi]\!] &= [\![\varphi]\!] \cup [\![\psi]\!]
\end{aligned}$$

(5) Does Mary sleep?

$$\begin{aligned}
?\,(\mathbf{sleep}\ \mathbf{m}) &= (\mathbf{sleep}\ \mathbf{m}) \vee (\neg\,(\mathbf{sleep}\ \mathbf{m})) \\
[\![?\,(\mathbf{sleep}\ \mathbf{m})]\!] &= \{\{\mathsf{M}\}, \emptyset\} \cup \{\{\mathsf{J}, \mathsf{A}\}, \{\mathsf{J}\}, \{\mathsf{A}\}, \emptyset\}
\end{aligned}$$

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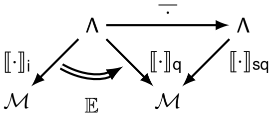
Looking for: **object language translation** $\overline{\cdot}$ such that

- $[\![\varphi]\!]_{\text{q}} = [\![\overline{\varphi}]\!]_{\text{sq}}$, for every φ



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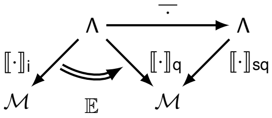

Syntactic translation

$$\begin{aligned}
\overline{\varphi \vee \psi} &= !(\overline{\varphi} \vee \overline{\psi}) \\
\overline{\exists x.\, \varphi} &= !(\exists x.\, \overline{\varphi}) \\
\overline{c} &= c \qquad \text{for the other constants} \\
\overline{t\,u} &= \overline{t}\,\overline{u} \\
\overline{\lambda x.\, t} &= \lambda x.\, \overline{t}
\end{aligned}$$

Conclusion

- **Conservative extension** from intensional semantics to inquisitive semantics

Future prospects

- Using inquisitivation to define higher-order inquisitive logic

[1] I. Ciardelli, J. Groenendijk, and F. Roelofsen. Inquisitive Semantics: A New Notion of Meaning. **Language and Linguistics Compass**, 7(9):459–476, 2013. ISSN 1749-818X. doi: 10.1111/lnc3.12037.

[2] I. Ciardelli, F. Roelofsen, and N. Theiler. Composing alternatives. **Linguistics and Philosophy**, 40(1):1–36, February 2017. ISSN 1573-0549. doi: 10.1007/s10988-016-9195-2.

[3] Ph. de Groote and M. Kanazawa. A Note on Intensionalization. **Journal of Logic, Language and Information**, 22(2):173–194, April 2013. ISSN 1572-9583. doi: 10.1007/s10849-013-9173-9.

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Interpretation of λ-terms:

$$\begin{aligned}
[\![x^{\alpha}]\!]_{\mathrm{i}\ \xi} &= \xi_{\alpha}(x) \\
[\![c^{\alpha}]\!]_{\mathrm{i}\ \xi} &= \mathcal{I}_{\alpha}(c) \\
[\![t^{\alpha \to \beta}\ u^{\alpha}]\!]_{\mathrm{i}\ \xi} &= [\![t^{\alpha \to \beta}]\!]_{\mathrm{i}\ \xi}([\![u^{\alpha}]\!]_{\mathrm{i}\ \xi}) \\
[\![\lambda x^{\alpha}.\, t^{\beta}]\!]_{\mathrm{i}\ \xi} &= a \in [\alpha]_{\mathrm{i}} \mapsto [\![t^{\beta}]\!]_{\mathrm{i}\ \xi[x^{\alpha} := a]}
\end{aligned}$$