# More efficient learning of strictly 2-local tier-based functions

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# Overview

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  - Previous approach
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# Notation

- The empty string is $\lambda$
- String concatenation is indicated with $\cdot$
  - $ab \cdot b = abb$
  - Omitted when context permits
- The exponent $-1$ represents string subtraction
  - $(ab)^{-1} \cdot abb = b$
- $\Sigma$ and $\Delta$ are the input and output alphabets
- $\Sigma^*$ is all strings of any length made using elements of $\Sigma$
- The *longest common prefix* of a stringset $S$ is $\texttt{lcp}(S)$
  - $\texttt{lcp}(\{a, aa, ab, aaa\}) = a$

# Notation

- For function $f$, its corresponding *prefix function* ($f^p$) is
  - $f^p(w) = \texttt{lcp}(f(w\Sigma^*))$
  - For example, if $f(w) = w \cdot a$ then $f^p(w) = w$
- The *contribution* of $x \in \Sigma^*$ given $w \in \Sigma^*$ with respect to the function $f$ is
  - $\texttt{cont}_f(x, w) = f^p(w)^{-1} \cdot f^p(wx)$
  - Essentially, the part of $f^p(wx)$ directly attributable to $x$
- The *contribution* of $\ltimes \notin \Sigma$ given $w \in \Sigma^*$ with respect to the function $f$ is
  - $\texttt{cont}_f(\ltimes, w) = f^p(w)^{-1} \cdot f(w)$
  - Essentially, the part of $f(w)$ that can't be directly attributed to input material

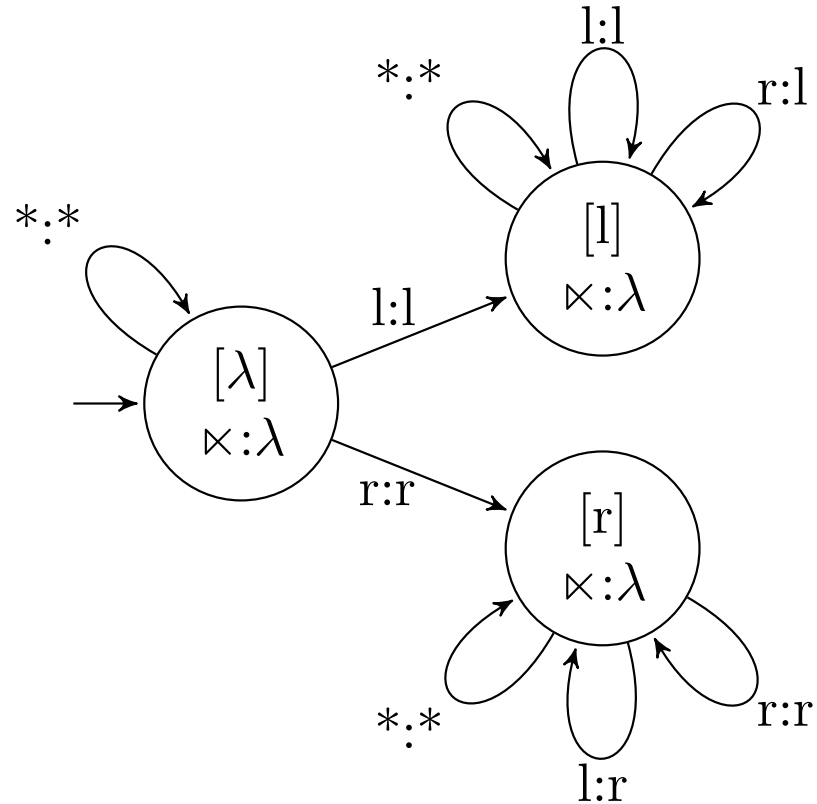
# Tier-based Strictly Local functions

- Functional analogues to the Tier-based Strictly Local (TSL) languages of Heinz et al. (2011)
- A *tier* is a subset of a given alphabet, containing only the "relevant" members
- Tiers let us create long-distance dependencies between tier elements by skipping over non-tier elements
- Given a tier $T$ and a string $w$, the tier projection $\pi_T(w)$ is $w$ with all non-members of $T$ removed
  - Given $T = \{l, r\}$, $\pi_T(lokiral) = lrl$
- We write $\texttt{suff}^n_T(w)$ for the $n$-long suffix of $\pi_T(w)$

# Tier-based Strictly Local functions

- A function is Input Tier-based Strictly k-Local ($\text{ITSL}_k$) iff there is a tier $T \subseteq \Sigma$ such that for all $x \in \Sigma^*$
  - $\texttt{suff}_T^{k-1}(w_1) = \texttt{suff}_T^{k-1}(w_2) \implies \texttt{cont}_f(x, w_1) = \texttt{cont}_f(x, w_2)$
- A function is Output Tier-based Strictly k-Local ($\text{OTSL}_k$) iff there is a tier $T \subseteq \Delta$ such that for all $x \in \Sigma^*$
  - $\texttt{suff}_T^{k-1}(f^p(w_1)) = \texttt{suff}_T^{k-1}(f^p(w_2)) \implies \texttt{cont}_f(x, w_1) = \texttt{cont}_f(x, w_2)$
- In other words, transformations depend entirely on the $k-1$ most recent input or output tier elements
- TSL functions can model many long-distance phonological processes such as vowel and consonant harmony (Andersson et al., 2020; Burness et al., 2021)

# TSL functions




OTSL$_2$ function for a simple pattern of liquid harmony

- /l/ → [r] / [r..._]
- /r/ → [l] / [l..._]

# Previous learning results

- Let $n$ be the size of the input data sample
- Strictly Local functions (of which TSL functions are a superclass) can be learned in $\mathcal{O}(n^2)$ time
  - Input Strictly Local Function Inference Algorithm (ISLFIA: Chandlee et al. 2014)
  - Output Strictly Local Function Inference Algorithm (OSLFIA: Chandlee et al. 2015)
- Subsequential functions (of which TSL functions are a subclass) can be learned in $\mathcal{O}(n^3)$ time
  - Onward Subsequential Transducer Inference Algorithm (OSTIA: Oncina et al. 1993)

# Previous learning results

- *When the tier is known*, TSL functions can be learned in $\mathcal{O}(n^2)$ time (Burness and McMullin, 2019)
- Identifying the tier is possible when $k = 2$ (Burness and McMullin, 2019)
- The method proposed by Burness and McMullin (2019) for this takes $\mathcal{O}(n^5)$ time
- We identified the bottlenecks responsible for the high time complexity, and will show how they can be eliminated to make tier identification run in $\mathcal{O}(n^2)$ time.

# Previous approach

- Our improved algorithm follows the overall architecture of the one in Burness and McMullin (2019)
- As an example target function, we will use symmetric sibilant harmony
  - $\Sigma = \{\text{s}, \text{ʃ}, \text{a}\}$
  - $\Delta = \{\text{s}, \text{ʃ}, \text{a}\}$
  - /s/ $\rightarrow$ [ʃ] if preceded by [ʃ] at any distance
  - /ʃ/ $\rightarrow$ [s] if preceded by [s] at any distance
  - $S = \{(w\ltimes, f(w)) \mid w \in \Sigma^{\leq 4}\}$
- The first thing that must be done: determine $f^p(w)$ for as many inputs $w$ as possible.

# Previous approach

- Start by adding all the input prefixes in $S$ to a set $X$
  - (sa⋉, sa) adds 'λ', 's', 'sa', and 'sa⋉' to $X$
- Then, for each $x \in X$ we check whether $x\sigma$ is also in $X$ for each $\sigma \in \Sigma \cup \{\ltimes\}$
- If so, we can calculate $f^p(x)$, so we add $x$ to another set $Y$.
  - For 's' $\in X$
    - 's⋉', 'sa', 'ss', 'sʃ'$\in X$
    - 's' is thus added to $Y$
- For each $y \in Y$, we determine $f^p(y) = \texttt{lcp}(\{u \mid (w, u) \in S$ and $y$ is a prefix of $w\})$
- This compiles a set $Z$ of pairs $(x, f^p(x))$ that is essentially a "snapshot" of $f^p$

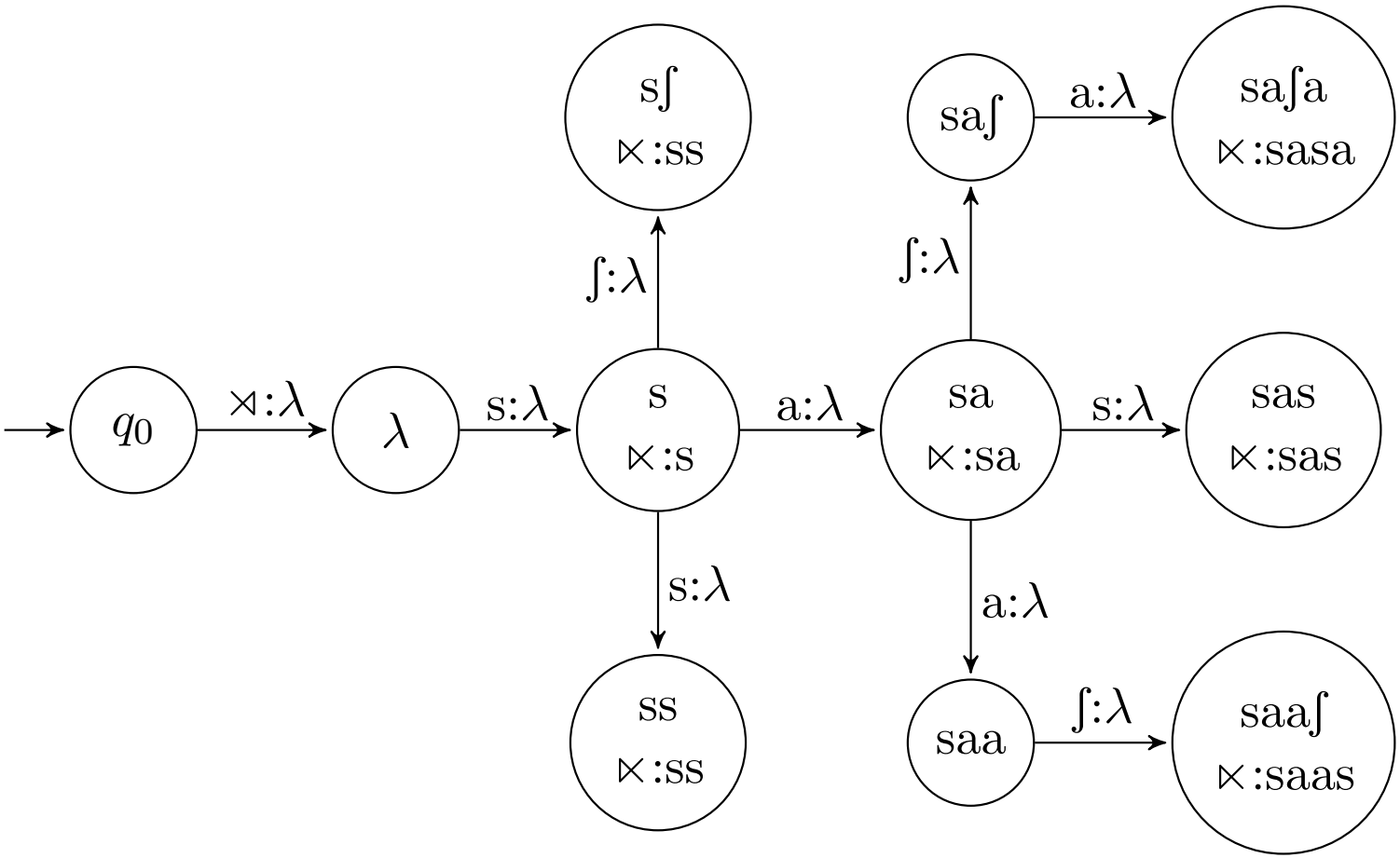
# Previous approach

- The process is inefficient because we are effectively performing nested reads of the sample
- Let $l$ be the number of training instances
  Let $m$ be the length of the longest input string
  Let $n$ be the summed length of the input strings
  - There can be up to $lm$ prefixes
  - For each, we calculate an `lcp` which can take up to $nm$ steps
  - Overall we have $\mathcal{O}(lm \cdot nm)$
- The same result can be obtained without nested reads by using a Prefix Tree Transducer (PTT)

# Prefix Tree Transducers

- A PTT corresponding to a sample generates all and only the pairs in the sample
- In their base form, they wait until the end of the input string is reached before outputting anything.
- The next slide shows the base PTT for the sample:
  - (s, s)
  - (ss, ss)
  - (sʃ, ss)
  - (sa, sa)
  - (sas, sas)
  - (saʃa, sasa)
  - (saaʃ, saas)

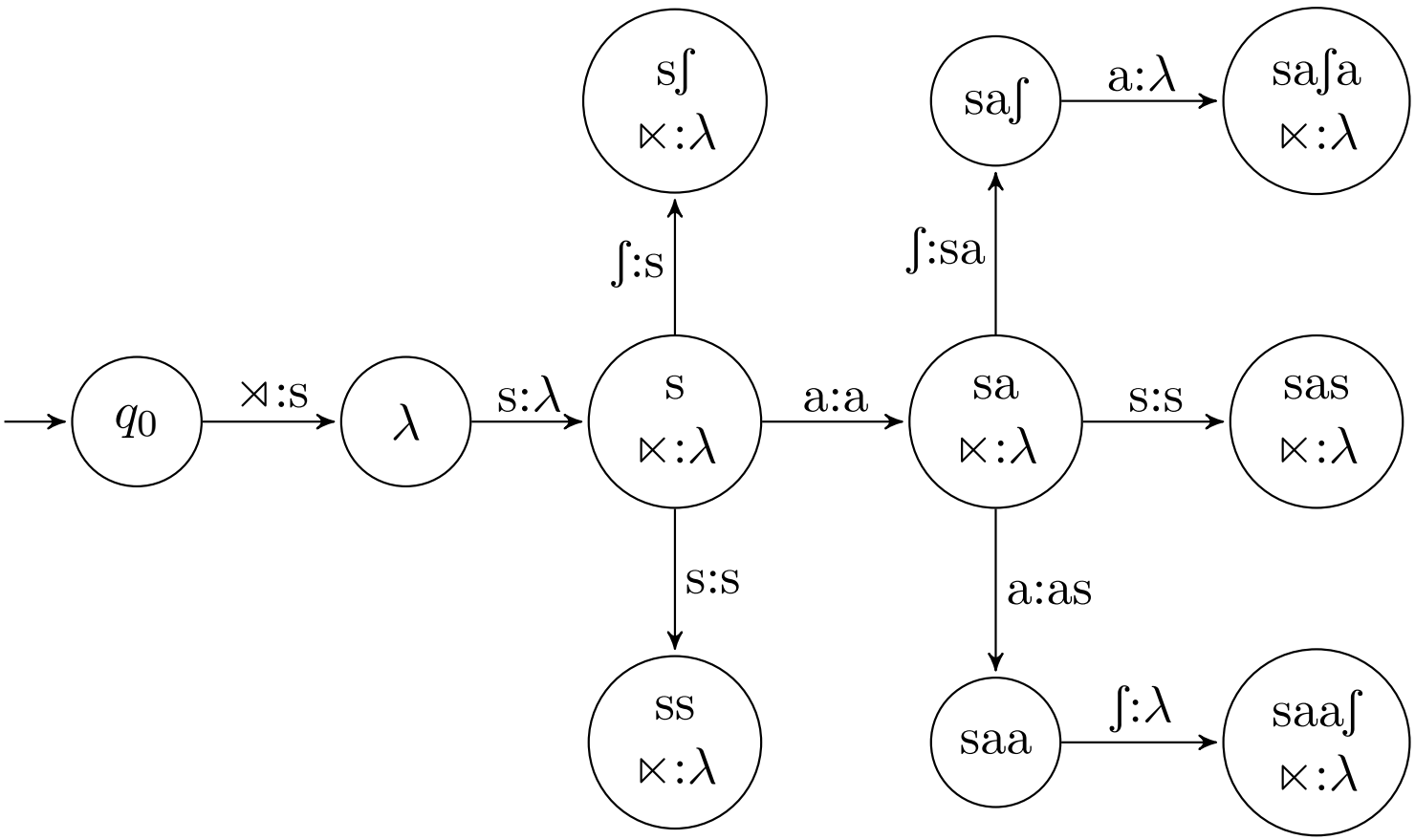
# Prefix Tree Transducers



# Onwarding

- A base PTT can be made *onward* so that it produces as much output as soon as possible
- For details on how to make a PTT onward, see chapter 18 of de la Higuera (2010)
- OSTIA and the ISLFIA (but not the OSLFIA) manipulate onward PTTs, merging their states until convergence
- For tier learning, we will use an onward PTT as a sort of oracle, consulting it for essential information
- The next slide shows the onward version of the prior PTT

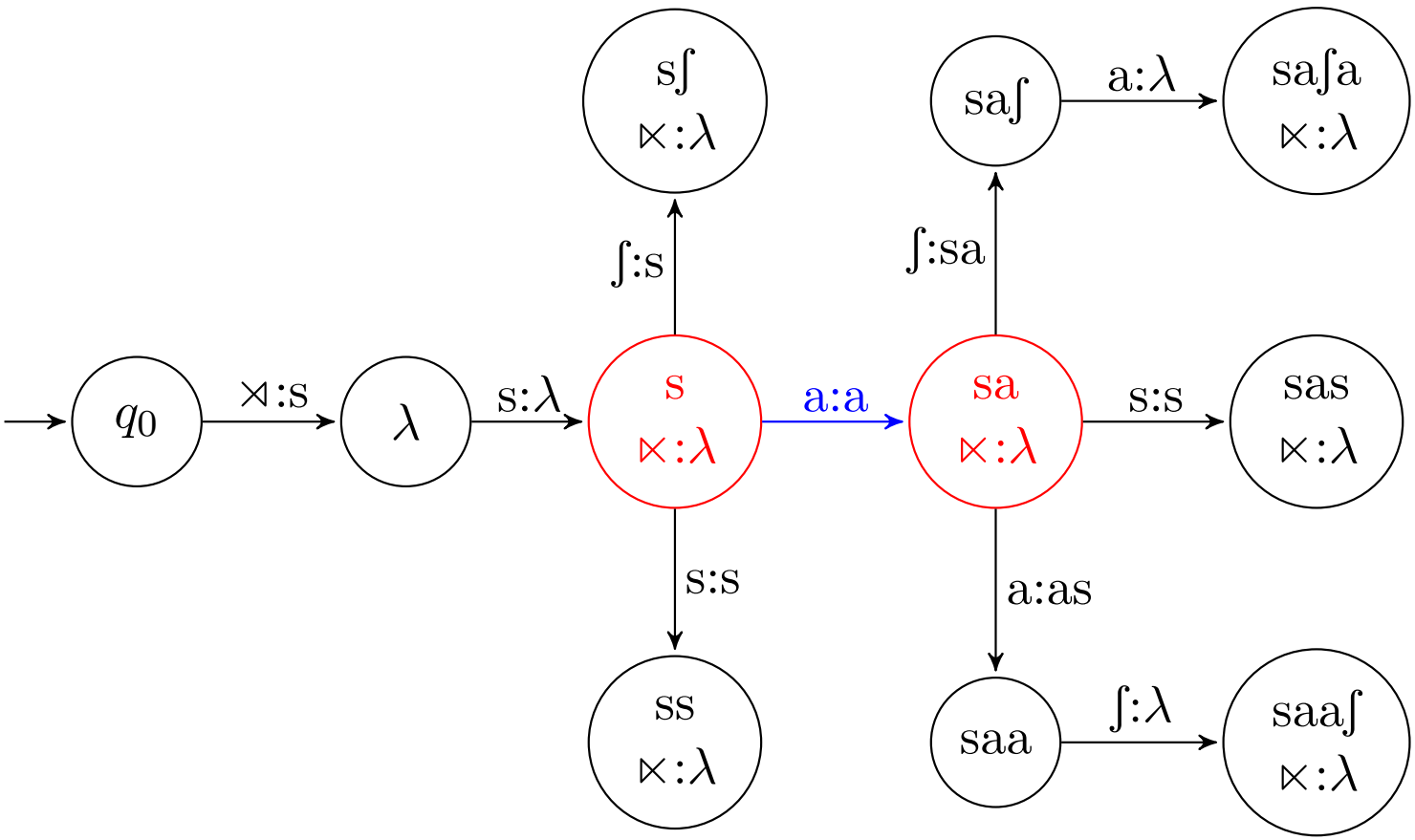
# Onwarding


$q_0$
⋊:s
λ
s:λ
s
⋉:λ
a:a
sa
⋉:λ
s:s
sas
⋉:λ
ʃ:s
sʃ
⋉:λ
s:s
ss
⋉:λ
ʃ:sa
saʃ
a:λ
saʃa
⋉:λ
a:as
saa
ʃ:λ
saaʃ
⋉:λ

# Current approach

- The key insight with regards to onward PTTs:
  - Given state $q$, if there is an outgoing transition for every possible input element (including word-end symbol $\ltimes$), then the PTT will have produced $f^p(q)$ upon landing in $q$
  - We call such states *supported states*
- Equally important corollaries:
  - The final transition out of supported $q$ produces $\texttt{cont}_f(\ltimes, q)$
  - A transition from supported $q$ to supported $q'$ for $\sigma$ produces $\texttt{cont}_f(\sigma, q)$

# Current approach



# Current approach

- We can compile the set $Z$ of pairs $(x, f^p(x))$ by sending the sample once through its onward PTT
- At each reading step, we check whether we are in a supported state.
- If yes, we add the corresponding pair to $Z$
- Onwarding the PTT takes time quadratic in the size of the sample, as does the just described process of checking the sample against the PTT
  - While $\Sigma$ is technically a constant for the learning problem, the runtime is linear in $|\Sigma|$
- Using a PTT reduces time complexity from $n^4$ to $n^2$

# Strategy overview

- Property 1: When a given $\text{TSL}_2$ function can be described using tier $A$ and using tier $B$, then it can also be described using tier $C = A \cup B$
  - Implies the existence of a largest *canonical* tier which properly contains any other tier that can be used
- Property 2: If we try to describe a $\text{TSL}_2$ function with a strict superset of its canonical tier, there will be evidence that a superfluous member can never be on any tier
  - There will never be such evidence for members of the canonical tier
- Consequence: We can start with the whole alphabet as the tier, and whittle it down as we find relevant evidence

# Previous approach

- We use the set $Z$ constructed by the previous portion of the algorithm (the "snapshot" of $f^p$)
- We cycle through the current tier hypothesis $H$ until all its elements get added to an auxiliary set $K$ (for "keep")
- To check whether $t \in H$ can be added to $K$
  - Gather all $(x, y) \in Z$ such that $\texttt{suff}^1_H(y) = t$
  - For each collected pair:
    - Check whether $(x\sigma, y') \in Z$ for each $\sigma \in \Sigma$
    - If so, calculate $\texttt{cont}_f(\sigma, x) = y^{-1} \cdot y'$
    - Check whether $(x\ltimes, y'') \in S$
    - If so, calculate add $\texttt{cont}_f(\ltimes, x) = y^{-1} \cdot y''$
  - Remove $t$ from $H$ if there are any mismatches
  - Add $t$ to $K$ otherwise

# Previous approach

- The process is inefficient because we are, again, effectively performing nested reads of the sample
- Let $l$ be the number of training instances
  Let $m$ be the length of the longest input string
  Let $n$ be the length of the longest output string
  - There can be up to $lm$ pairs in $Z$ (the "snapshot" of $f^p$)
  - The initial search takes up to $m$ steps per pair
  - We run an additional search through the "found" pairs, of which there can be up to $lm$
  - The additional search takes up to $n$ steps per pair
  - Overall we have $\mathcal{O}(lm(m + lmn))$
- As above, the desired result can be obtained without nested reads by using a Prefix Tree Transducer (PTT)

# Current approach

- We do not need to re-calculate contributions directly from the sample every time
- Instead, we can cycle through the transitions $(q, \sigma, u, q')$ in the onward PTT:
  - Check whether $q$ is supported (i.e., $\exists (q, r) \in Z$)
  - If yes, check whether $q'$ is supported (i.e., $\exists (q', r') \in Z$)
  - If yes, put the transition in a bin tied to the input element $\sigma$ and the tier element $\texttt{suff}^1_H(q)$
  - If any bins contain mismatches, the corresponding tier element gets removed from the tier hypothesis

# Current approach

- Let $l$ be the number of training instances
  Let $m$ be the length of the longest output string
  Let $n$ be the summed length of the input strings
  - There are $l + n$ transitions in the onward PTT
  - Checking whether a state is supported takes at most $n$ steps
  - Getting the tier suffix of a state label takes at most $m$ steps
  - Overall we have $\mathcal{O}((l + n)(m + n))$
  - While $\Sigma$ and $\Delta$ are technically constants for the learning problem, the runtime is linear in $|\Sigma|$ and quadratic in $|\Delta|$
- Using a PTT reduces time complexity from $n^5$ to $n^2$

# SL versus subsequential

- Using ISL and OSL functions instead of subsequential functions sacrifices expressiveness, but comes with a gain in learning efficiency.
- Where the runtime of OSTIA is cubic, the runtimes of the ISLFIA and OSLFIA are both quadratic
- The time improvement of the ISLFIA over OSTIA is analogous to our elimination of nested reading
  - OSTIA can undo state merges whereas the ISLFIA cannot

# Strictly local tier-based functions

- For tier-based functions, the same time-expressiveness trade-off previously applied only when the tier was known
- Burness and McMullin (2019) showed that the tier of a $\text{TSL}_2$ function could be learned in quintic time
  - We have shown that this can be reduced to quadratic time
- We have focused on the $\text{OTSL}_2$ class throughout; only slight modifications are needed for $\text{ITSL}_2$ functions
- Other slight modifications can learn particular classes of multi-tiered functions (Burness and McMullin, 2020)

# Other advantages

- Subsequential computation is sufficiently expressive for
  - Vowel harmony (Heinz and Lai, 2013)
  - Consonant harmony (Luo, 2017)
  - Consonant dissimilation (Payne, 2017)
- TSL functions, however, provide more intuitive analyses (Andersson et al., 2020; Burness et al., 2021)
- TSL functions exclude un-phonological behaviours like modulo counting and minimum distance requirements (Burness et al., 2021)

# Future directions

- Tier learning for functions with $k > 2$
  - The tier of tier-based *languages* can be learned for any $k$ (Jardine and McMullin, 2017; Lambert, 2021)
- Online learning instead of batch learning
  - Tier-based languages can be learned in batch (Jardine and McMullin, 2017) or online (Lambert, 2021)
- Tier learning for partial functions
  - Strategy assumes a total function
  - Domain and range information allow OSTIA to learn partial functions (Oncina and Varó, 1996; Castellanos et al., 1998)

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